Reconstructing interacting entropy-corrected holographic scalar field models of dark energy in non-flat universe

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Abstract

Here we consider the entropy-corrected version of the holographic dark energy model in the non-flat universe. We obtain the equation of state parameter in the presence of interaction between dark energy and dark matter. Moreover, we reconstruct the potential and the dynamics of the quintessence, tachyon, K-essence and dilaton scalar field models according to the evolutionary behavior of the interacting entropy-corrected holographic dark energy model.

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1 Introduction

The present acceleration of the universe expansion has been well established through numerous and complementary cosmological observations [1]. A component which is responsible for this accelerated expansion usually dubbed "dark energy" (DE). However, the nature of DE is still unknown, and people have proposed some candidates to describe it (for a good review see [2, 3] and references therein).

The holographic DE (HDE) is one of interesting DE candidates which was proposed based on the holographic principle [4]. According to the holographic principle, the number of degrees of freedom of a physical system should scale with its bounding area rather than with its volume [5] and it should be constrained by an infrared cut-off [6]. By applying the holographic principle to cosmology, one can obtain the upper bound of the entropy contained in the universe [7]. Following this line, Li [8] suggested the following constraint on its energy density $\rho_\Lambda \leq 3c^2M_P^2L^{-2}$, the equality sign holding only when the holographic bound is saturated. In this expression c is a numerical constant, L denotes the IR cut-off radius and $M_P = (8\pi G)^{-1/2}$ is the reduced Planck Mass. The HDE models have been studied widely in the literature [9, 10, 11, 12]. Obviously, in the derivation of HDE, the black hole entropy $S_{\rm BH}$ plays an important role. As is well known, usually, $S_{\rm BH} = A/(4G)$, where $A \sim L^2$ is the area of horizon. However, in the literature, this entropy-area relation can be modified to [13]

$$S_{\rm BH} = \frac{A}{4G} + \tilde{\alpha}\ln\frac{A}{4G} + \tilde{\beta}, \tag{1}$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ are dimensionless constants of order unity. These corrections can appear in the black hole entropy in loop quantum gravity (LQG) [14]. They can also be due to thermal equilibrium fluctuation, quantum fluctuation, or mass and charge fluctuations (for review see [14] and references therein). Using the corrected entropy-area relation (1), the energy density of the entropy-corrected HDE (ECHDE) can be obtained as [14]

$$\rho_\Lambda = 3c^2M_P^2L^{-2} + \alpha L^{-4}\ln(M_P^2L^2) + \beta L^{-4}, \tag{2}$$

where α and β are dimensionless constants of order unity. In the special case $\alpha = \beta = 0$, the above equation yields the well-known HDE density. Since the last two terms in Eq. (2) can be comparable to the first term only when L is very small, the corrections make sense only at the early stage of the universe. When the universe becomes large, ECHDE reduces to the ordinary HDE [14].

Reconstructing the holographic and agegraphic scalar field models of DE is one of interesting issue which has been investigated in the literature [15, 16, 17]. The holographic and the agegraphic DE models are originated from some considerations of the features of the quantum theory of gravity. On the other hand, the scalar field models (such as quintessence, tachyon, K-essence and dilaton) are often regarded as an effective description of an underlying theory of DE [17]. The scalar field models can mimic cosmological constant at the present epoch and can give rise to other observed values of the equation of state parameter ω (recent data indicate that ω lies in a narrow strip around $\omega = \omega_\Lambda = -1$ and is consistent with being below this value) [18]. They can also alleviate the fine tuning and coincidence problems [18]. Therefore it becomes meaningful to reconstruct the scalar

field models from some DE models possessing some significant features of the LQG theory, such as ECHDE and entropy-corrected agegraphic DE (ECADE) models.

An interesting feature of entropy-corrected DE is that it permits successive acceleration-deceleration phase transitions. Moreover the cosmic coincidence problem is resolved and the universe eventually tends to de Sitter expansion [19]. Here our aim is to investigate the correspondence between the entropy-corrected version of the interacting HDE model with the quintessence, tachyon, K-essence and dilaton scalar field models in the non-flat universe. These correspondences are essential to understand the connection of various scalar field models of DE with the ECHDE. This paper is organized as follows. In Section 2, we obtain the equation of state parameter for the interacting ECHDE model in a non-flat universe. In Sections 3-6, we suggest a correspondence between the interacting ECHDE and the quintessence, tachyon, K-essence and dilaton scalar field models in the presence of a spatial curvature. We reconstruct the potentials and the dynamics for these scalar field models, which describe accelerated expansion of the universe. Section 7 is devoted to conclusions.

2 Interacting ECHDE and DM in non-flat universe

Within the framework of the standard FRW cosmology,

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\Omega^2\right), \tag{3}$$

for the non-flat FRW universe containing the ECHDE and DM, the first Friedmann equation takes the form

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_P^2}\,(\rho_\Lambda + \rho_{\mathrm{m}}), \tag{4}$$

where $k = 0, 1, -1$ represent a flat, closed and open FRW universe, respectively. Also ρ_Λ and ρ_{m} are the energy density of ECHDE and DM, respectively. Observational evidences have implied that our universe is not a perfectly flat universe and that it possesses a small positive curvature [20]. Besides, as usually believed, an early inflation era leads to a flat universe. This is not a necessary consequence if the number of e-foldings is not very large [21]. Additionally the parameter Ω_k (discussed below) represents the contribution in the total energy density from the spatial curvature and is constrained as $-0.0175 < \Omega_k < 0.0085$ with 95% confidence level by current observations [22]. It has been shown that a non-zero positive curvature parameter k allows for a bounce, thereby preventing the cosmic singularities without violating the null energy condition $\rho + p \geq 0$ [23].

From Eq. (4), we can write

$$\Omega_{\mathrm{m}} + \Omega_\Lambda = 1 + \Omega_k, \tag{5}$$

where we have used the following definitions

$$\Omega_{\mathrm{m}} = \frac{\rho_{\mathrm{m}}}{\rho_{\mathrm{cr}}} = \frac{\rho_{\mathrm{m}}}{3M_P^2H^2}, \qquad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{\mathrm{cr}}} = \frac{\rho_\Lambda}{3M_P^2H^2}, \qquad \Omega_k = \frac{k}{a^2H^2}. \tag{6}$$

The recent observational evidence provided by the galaxy cluster Abell A586 supports the interaction between DE and DM [24]. This motivates us to consider the interaction between ECHDE and DM. Hence ρ_Λ and $\rho_{\rm m}$ do not conserve separately and the energy conservation equations for ECHDE and DM are

$$\dot{\rho}_\Lambda + 3H(1+\omega_\Lambda)\rho_\Lambda = -Q, \tag{7}$$

$$\dot{\rho}_{\rm m} + 3H\rho_{\rm m} = Q, \tag{8}$$

where following [25], we choose $Q = \Gamma\rho_\Lambda$ as an interaction term and $\Gamma = 3b^2H(\frac{1+\Omega_k}{\Omega_\Lambda})$ is the decay rate of the ECHDE component into DM with a coupling constant b^2. Although this expression for the interaction term may look purely phenomenological but different Lagrangians have been proposed in support of it [26].

Note that in Eq. (2), taking L as the size of the current universe, for instance, the Hubble scale, the resulting energy density is comparable to the present day DE. However, as found by Hsu [27], in that case, the evolution of the DE is the same as that of DM (dust matter), and therefore it cannot drive the universe to accelerated expansion. The same appears if one chooses the particle horizon of the universe as the length scale L [8]. To obtain an accelerating universe, Li [8] proposed that for a flat universe, L should be the future event horizon $R_{\rm h}$ and Huang and Li [21] argued that for the non-flat case, the IR cut-off L should be defined as

$$L = a\frac{\sin n(\sqrt{|k|}y)}{\sqrt{|k|}}, \tag{9}$$

where

$$y = \frac{R_{\rm h}}{a} = \int_t^\infty \frac{{\rm d}t}{a} = \int_0^r \frac{{\rm d}r}{\sqrt{1-kr^2}}. \tag{10}$$

Here $R_{\rm h}$ is the radial size of the event horizon measured in the r direction and L is the radius of the event horizon measured on the sphere of the horizon [21]. For a flat universe, $L = R_{\rm h}$. The last integral in Eq. (10) has the explicit form as

$$\int_0^r \frac{{\rm d}r}{\sqrt{1-kr^2}} = \frac{1}{\sqrt{|k|}}\sin n^{-1}\left(\sqrt{|k|}r\right) = \begin{cases} \sin^{-1} r, & k=1, \\ r, & k=0, \\ \sinh^{-1} r, & k=-1. \end{cases} \tag{11}$$

From definition $\rho_\Lambda = 3M_P^2H^2\Omega_\Lambda$ and using Eq. (2), we get

$$L = \frac{c}{H}\left(\frac{\gamma_c}{\Omega_\Lambda}\right)^{1/2}, \tag{12}$$

where

$$\gamma_c = 1 + \frac{1}{3c^2M_P^2L^2}\Big[\alpha\ln{(M_P^2L^2)} + \beta\Big]. \tag{13}$$

Taking time derivative of Eq. (9) and using (12) yields

$$\dot{L} = c\left(\frac{\gamma_c}{\Omega_\Lambda}\right)^{1/2} - \cos n\left(\sqrt{|k|}y\right), \tag{14}$$

where

$$\cos n\left(\sqrt{|k|}y\right)=\begin{cases}\cos y, & k=1,\\ 1, & k=0,\\ \cosh y, & k=-1.\end{cases}\tag{15}$$

Taking time derivative of Eq. (2) and using (12) and (14), one can obtain

$$\dot{\rho_\Lambda}=\left(\frac{2H\rho_\Lambda}{\gamma_c}\right)\left[1-2\gamma_c+\frac{\alpha H^2}{3c^2M_P^2}\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)\right]\left[1-\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)^{1/2}\cos n\left(\sqrt{|k|}y\right)\right].\tag{16}$$

Substituting Eq. (16) in (7) gives the equation of state (EoS) parameter of the interacting ECHDE as

$$\begin{aligned}\omega_\Lambda=&-1-b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right)\\ &-\frac{2}{3\gamma_c}\left[1-2\gamma_c+\frac{\alpha H^2}{3c^2M_P^2}\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)\right]\left[1-\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)^{1/2}\cos n\left(\sqrt{|k|}y\right)\right].\end{aligned}\tag{17}$$

Note that as we already mentioned, at the very early stage when the universe undergoes an inflation phase, the correction terms in the ECHDE density (2) become important. After the end of the inflationary phase, the universe subsequently enters in the radiation and then matter dominated eras. In these two epochs, since the universe is much larger, the entropy-corrected terms to ECHDE, namely the last two terms in Eq. (2), can be safely ignored. Therefore if we set $\alpha=\beta=0$, then from Eq. (13) $\gamma_c=1$ and Eq. (17) recovers the EoS parameter of the ordinary HDE [28]

$$\omega_\Lambda=-\frac{1}{3}-b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right)-\frac{2\sqrt{\Omega_\Lambda}}{3c}\cos n\left(\sqrt{|k|}y\right).\tag{18}$$

In next sections, we suggest a correspondence between the interacting ECHDE model with the quintessence, tachyon, K-essence and dilaton scalar field models in the non-flat universe.

3 Entropy-corrected holographic quintessence model

Quintessence is described by an ordinary time dependent and homogeneous scalar field ϕ which is minimally coupled to gravity, but with a particular potential $V(\phi)$ that leads to the accelerating universe. The action for quintessence is given by [3]

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)\right].\tag{19}$$

The energy momentum tensor of the field is derived by varying the action (19) with respect to $g^{\mu\nu}$:

$$T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}},\tag{20}$$

which gives

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right]. \tag{21}$$

The energy density and pressure of the quintessence scalar field ϕ are as follows

$$\rho_Q = -T_0^0 = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{22}$$

$$p_Q = T_i^i = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{23}$$

The EoS parameter for the quintessence scalar field is given by

$$\omega_Q = \frac{p_Q}{\rho_Q} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}. \tag{24}$$

From (24) for $\omega_Q < -1/3$, we find that the universe accelerates when $\dot{\phi}^2 < V(\phi)$.

Here we establish the correspondence between the interacting ECHDE scenario and the quintessence DE model, then equating Eq. (24) with the EoS parameter of interacting ECHDE (17), $\omega_Q = \omega_\Lambda$, and also equating Eq. (22) with (2), $\rho_Q = \rho_\Lambda$, we have

$$\dot{\phi}^2 = (1+\omega_\Lambda)\rho_\Lambda, \tag{25}$$

$$V(\phi) = \frac{1}{2}(1-\omega_\Lambda)\rho_\Lambda. \tag{26}$$

Substituting Eqs. (2) and (17) into Eqs. (25) and (26), one can obtain the kinetic energy term and the quintessence potential energy as follows

$$\dot{\phi}^2 = 3M_P^2H^2\Omega_\Lambda\Bigg(b^2\Big(\frac{1+\Omega_k}{\Omega_\Lambda}\Big) + \frac{2}{3\gamma_c}\Big[1-2\gamma_c+\frac{\alpha H^2}{3c^2M_P^2}\Big(\frac{\Omega_\Lambda}{c^2\gamma_c}\Big)\Big]\Big[1-\Big(\frac{\Omega_\Lambda}{c^2\gamma_c}\Big)^{1/2}\cos n\Big(\sqrt{|k|}y\Big)\Big]\Bigg), \tag{27}$$

$$V(\phi) = 3M_P^2H^2\Omega_\Lambda\Bigg(1+\frac{b^2}{2}\Big(\frac{1+\Omega_k}{\Omega_\Lambda}\Big) + \frac{1}{3\gamma_c}\Big[1-2\gamma_c+\frac{\alpha H^2}{3c^2M_P^2}\Big(\frac{\Omega_\Lambda}{c^2\gamma_c}\Big)\Big]\Big[1-\Big(\frac{\Omega_\Lambda}{c^2\gamma_c}\Big)^{1/2}\cos n\Big(\sqrt{|k|}y\Big)\Big]\Bigg). \tag{28}$$

From Eqs. (27) one can obtain the evolutionary form of the quintessence scalar field as

$$\phi(a)-\phi(a_0) = M_P\int_{a_0}^{a}\Bigg(3b^2(1+\Omega_k) + \frac{2\Omega_\Lambda}{\gamma_c}\Big[1-2\gamma_c+\frac{\alpha H^2}{3c^2M_P^2}\Big(\frac{\Omega_\Lambda}{c^2\gamma_c}\Big)\Big]\Big[1-\Big(\frac{\Omega_\Lambda}{c^2\gamma_c}\Big)^{1/2}\cos n\Big(\sqrt{|k|}y\Big)\Big]\Bigg)^{1/2}\frac{\mathrm{d}a}{a}, \tag{29}$$

where a_0 is the scale factor at the present time.

The above integral cannot be taken analytically. But during the early inflation era when the correction terms make sense in the ECHDE density (2), the Hubble parameter H is constant and $a = a_0 e^{Ht}$. Hence the Hubble horizon H^{-1} and the future event horizon $R_{\rm h} = a \int_t^\infty \frac{{\rm d}t}{a}$ will coincide i.e. $R_{\rm h} = H^{-1} = \text{const.}$ On the other hand, since an early inflation era leads to a flat universe we have $L = R_{\rm h} = H^{-1} = \text{const.}$ Also from Eqs. (12) and (15) we have $\frac{\Omega_\Lambda}{c^2\gamma_c} = 1$ and $\cos n\left(\sqrt{|k|}y\right) = 1$. Therefore during the early inflation era, Eq. (29) reduces to

$$\phi(a) = \phi(a_0) + \sqrt{3}\, bM_P \ln\left(\frac{a}{a_0}\right). \tag{30}$$

For the late-time universe, i.e. $\Omega_\Lambda = 1$ and $\Omega_k = 0$, the universe becomes large and ECHDE reduces to the ordinary HDE. In this case $L = R_{\rm h} \neq H^{-1}$ and $H \neq \text{const.}$ Now by setting $\gamma_c = 1(\alpha = \beta = 0)$ and $\cos n\left(\sqrt{|k|}y\right) = 1$, the Hubble parameter from Eqs. (12) and (14) can be obtained as

$$H = \frac{H_0}{1 + \left(\frac{c-1}{c}\right) H_0 (t - t_0)}, \tag{31}$$

where H_0 is the Hubble parameter at the present time. After integration of Eq. (31) with respect to t, the scale factor can be obtained as

$$a = a_0 \left[1 + \left(\frac{c-1}{c}\right) H_0 (t - t_0)\right]^{\frac{c}{c-1}}. \tag{32}$$

Using the above relation, one can rewrite Eq. (31) as

$$H = H_0 \left(\frac{a}{a_0}\right)^{\frac{1-c}{c}}. \tag{33}$$

Finally for the late-time universe, Eq. (29) yields

$$\phi(a) = \phi(a_0) + M_P \left[3b^2 - 2\left(1 - \frac{1}{c}\right)\right]^{1/2} \ln\left(\frac{a}{a_0}\right). \tag{34}$$

4 Entropy-corrected holographic tachyon model

In recent years, a huge interest has been devoted in studying the inflationary model with the help of tachyon field. The tachyon field associated with unstable D-branes might be responsible for cosmological inflation in the early evolution of the universe, due to tachyon condensation near the top of the effective scalar potential [29]. Also the tachyonic matter could suggests some new form of DM at late epoch [30]. The tachyon field has emerged as a possible source of the DE. A rolling tachyon has an interesting EoS whose parameter smoothly interpolates between -1 and 0 [31]. This discovery motivated to take DE as the dynamical quantity, i.e. a variable cosmological constant and model inflation using tachyons. The effective Lagrangian density of tachyon matter is given by [32]

$$\mathcal{L} = -V(\phi)\sqrt{1 + \partial_\mu \phi \partial^\mu \phi}. \tag{35}$$

The energy density and pressure for the tachyon field are as following [32]

$$\rho_T = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}, \tag{36}$$

$$p_T = -V(\phi)\sqrt{1-\dot{\phi}^2}, \tag{37}$$

where $V(\phi)$ is the tachyon potential. The EoS parameter for the tachyon scalar field is obtained as

$$\omega_T = \frac{p_T}{\rho_T} = \dot{\phi}^2 - 1. \tag{38}$$

If we establish the correspondence between the ECHDE and tachyon DE, then equating Eq. (38) with the EoS parameter of interacting ECHDE (17), $\omega_T = \omega_\Lambda$, and also equating Eq. (36) with (2), $\rho_T = \rho_\Lambda$, we obtain

$$\dot{\phi}^2 = b^2\Big(\frac{1+\Omega_k}{\Omega_\Lambda}\Big) + \frac{2}{3\gamma_c}\Big[1-2\gamma_c+\frac{\alpha H^2}{3c^2M_P^2}\Big(\frac{\Omega_\Lambda}{c^2\gamma_c}\Big)\Big]\Big[1-\Big(\frac{\Omega_\Lambda}{c^2\gamma_c}\Big)^{1/2}\cos n\Big(\sqrt{|k|}y\Big)\Big], \tag{39}$$

$$\begin{aligned} V(\phi) = 3M_P^2H^2\Omega_\Lambda \Bigg(1+b^2\Big(\frac{1+\Omega_k}{\Omega_\Lambda}\Big) \\ +\frac{2}{3\gamma_c}\Big[1-2\gamma_c+\frac{\alpha H^2}{3c^2M_P^2}\Big(\frac{\Omega_\Lambda}{c^2\gamma_c}\Big)\Big]\Big[1-\Big(\frac{\Omega_\Lambda}{c^2\gamma_c}\Big)^{1/2}\cos n\Big(\sqrt{|k|}y\Big)\Big]\Bigg)^{1/2}. \end{aligned} \tag{40}$$

From Eq. (39), one can obtain the evolutionary form of the tachyon scalar field as

$$\begin{aligned} \phi(a)-\phi(a_0) = \int_{a_0}^{a}\frac{\mathrm{d}a}{Ha}\Bigg(b^2\Big(\frac{1+\Omega_k}{\Omega_\Lambda}\Big) \\ +\frac{2}{3\gamma_c}\Big[1-2\gamma_c+\frac{\alpha H^2}{3c^2M_P^2}\Big(\frac{\Omega_\Lambda}{c^2\gamma_c}\Big)\Big]\Big[1-\Big(\frac{\Omega_\Lambda}{c^2\gamma_c}\Big)^{1/2}\cos n\Big(\sqrt{|k|}y\Big)\Big]\Bigg)^{1/2}. \end{aligned} \tag{41}$$

During the early inflation era ($L = R_\mathrm{h} = H^{-1} = \mathrm{const.}$), Eq. (41) yields

$$\phi(a) = \phi(a_0) + \frac{b}{H\sqrt{\Omega_\Lambda}}\ln\Big(\frac{a}{a_0}\Big), \tag{42}$$

where

$$\Omega_\Lambda = c^2 + \frac{H^2}{3M_P^2}\Big[\alpha\ln(M_P^2H^{-2})+\beta\Big]. \tag{43}$$

For the late-time universe, i.e. $\Omega_\Lambda = 1$, $\Omega_k = 0$ and $\gamma_c = 1(\alpha=\beta=0)$, using Eq. (33) one can take the integral (41) as

$$\phi(a) = \phi(a_0) + \frac{\Big[b^2-\frac{2}{3}\Big(1-\frac{1}{c}\Big)\Big]^{1/2}}{H_0\Big(1-\frac{1}{c}\Big)}\Bigg[\Big(\frac{a}{a_0}\Big)^{1-\frac{1}{c}}-1\Bigg]. \tag{44}$$

5 Entropy-corrected holographic K-essence model

A model in which the kinetic energy term of the scalar field appears in the Lagrangian in a non-canonical way is termed the K-essence model. Such fields were originally used to model inflation, a scenario called K-inflation [33]. Moreover the stable tracker solutions for K-essence have been obtained i.e. solutions which start from arbitrary initial conditions and reach to the same final state of cosmic acceleration [34]. The purpose of introducing K-essence is to provide a dynamical explanation which does not require the fine-tuning of initial conditions. It is also possible to have a situation where the accelerated expansion of the universe arises out of modifications to the kinetic energy of the scalar fields. The K-essence is described by a general scalar field action which is a function of ϕ and $\chi = \dot{\phi}^2/2$, and is given by [35, 36]

$$S = \int \mathrm{d}^4x\sqrt{-\mathrm{g}}\; p(\phi, \chi), \tag{45}$$

where $p(\phi, \chi)$ corresponds to a pressure density as

$$p(\phi, \chi) = f(\phi)(-\chi + \chi^2), \tag{46}$$

and the energy density of the field ϕ is

$$\rho(\phi, \chi) = f(\phi)(-\chi + 3\chi^2). \tag{47}$$

The EoS parameter for the K-essence scalar field is obtained as

$$\omega_K = \frac{p(\phi, \chi)}{\rho(\phi, \chi)} = \frac{\chi - 1}{3\chi - 1}. \tag{48}$$

Equating Eq. (48) with the EoS parameter (17), $\omega_K = \omega_\Lambda$, we find the solution for χ

$$\chi = \frac{2 + b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right) + \frac{2}{3\gamma_c}\left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2M_P^2}\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)\right]\left[1 - \left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)^{1/2}\cos n\left(\sqrt{|k|}y\right)\right]}{4 + 3b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right) + \frac{2}{\gamma_c}\left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2M_P^2}\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)\right]\left[1 - \left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)^{1/2}\cos n\left(\sqrt{|k|}y\right)\right]}. \tag{49}$$

Using $\dot{\phi}^2 = 2\chi$ and (49), we obtain the evolutionary form of the K-essence scalar field as

$$\phi(a) = \phi(a_0)$$
$$+ \int_{a_0}^{a} \frac{\mathrm{d}a}{Ha}\left(\frac{1 + \frac{b^2}{2}(\frac{1+\Omega_k}{\Omega_\Lambda}) + \frac{1}{3\gamma_c}\left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2M_P^2}\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)\right]\left[1 - \left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)^{1/2}\cos n\left(\sqrt{|k|}y\right)\right]}{1 + \frac{3b^2}{4}(\frac{1+\Omega_k}{\Omega_\Lambda}) + \frac{1}{2\gamma_c}\left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2M_P^2}\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)\right]\left[1 - \left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)^{1/2}\cos n\left(\sqrt{|k|}y\right)\right]}\right)^{1/2}. \tag{50}$$

During the early inflation era, Eq. (50) reduces to

$$\phi(a) = \phi(a_0) + \frac{1}{H}\left(\frac{4\Omega_\Lambda + 2b^2}{4\Omega_\Lambda + 3b^2}\right)^{1/2} \ln\left(\frac{a}{a_0}\right), \tag{51}$$

where Ω_Λ is given by Eq. (43).

For the late-time universe, i.e. $L = R_{\mathrm{h}} \neq H^{-1}$ and $H \neq$ const., using Eq. (33) one can take the integral (50) as

$$\phi(a) = \phi(a_0) + \left(\frac{1 + \frac{b^2}{2} - \frac{1}{3}(1 - \frac{1}{c})}{1 + \frac{3b^2}{4} - \frac{1}{2}(1 - \frac{1}{c})}\right)^{1/2} \frac{\left[\left(\frac{a}{a_0}\right)^{1-\frac{1}{c}} - 1\right]}{H_0(1 - \frac{1}{c})}. \tag{52}$$

6 Entropy-corrected holographic dilaton model

The process of compactification of the string theory from higher to four dimensions introduces the scalar dilaton field which is coupled to curvature invariants. The coefficient of the kinematic term of the dilaton can be negative in the Einstein frame, which means that the dilaton behaves as a phantom-type scalar field. The pressure (Lagrangian) density and the energy density of the dilaton DE model is given by [37]

$$p_D = -\chi + c' e^{\lambda\phi}\chi^2, \tag{53}$$

$$\rho_D = -\chi + 3c' e^{\lambda\phi}\chi^2, \tag{54}$$

where c' and λ are positive constants and $\chi = \dot{\phi}^2/2$. The EoS parameter for the dilaton scalar field is given by

$$\omega_D = \frac{p_D}{\rho_D} = \frac{-1 + c' e^{\lambda\phi}\chi}{-1 + 3c' e^{\lambda\phi}\chi}. \tag{55}$$

Equating Eq. (55) with the EoS parameter (17), $\omega_D = \omega_\Lambda$, we find the following solution

$$c' e^{\lambda\phi}\chi = \frac{2 + b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right) + \frac{2}{3\gamma_c}\left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_P^2}\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)\right]\left[1 - \left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)^{1/2}\cos n\left(\sqrt{|k|}y\right)\right]}{4 + 3b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right) + \frac{2}{\gamma_c}\left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_P^2}\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)\right]\left[1 - \left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)^{1/2}\cos n\left(\sqrt{|k|}y\right)\right]}, \tag{56}$$

then using $\dot{\phi}^2 = 2\chi$, we obtain

$$e^{\frac{\lambda\phi}{2}}\dot{\phi} = \left(\frac{4 + 2b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right) + \frac{4}{3\gamma_c}\left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_P^2}\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)\right]\left[1 - \left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)^{1/2}\cos n\left(\sqrt{|k|}y\right)\right]}{c'\left(4 + 3b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right) + \frac{2}{\gamma_c}\left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_P^2}\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)\right]\left[1 - \left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)^{1/2}\cos n\left(\sqrt{|k|}y\right)\right]\right)}\right)^{1/2}. \tag{57}$$

Integrating with respect to a, we get

$$e^{\frac{\lambda\phi(a)}{2}} = e^{\frac{\lambda\phi(a_0)}{2}}$$
$$+\frac{\lambda}{2\sqrt{c'}}\int_{a_0}^{a}\frac{\mathrm{d}a}{Ha}\left(\frac{4 + 2b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right) + \frac{4}{3\gamma_c}\left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_P^2}\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)\right]\left[1 - \left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)^{1/2}\cos n\left(\sqrt{|k|}y\right)\right]}{4 + 3b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right) + \frac{2}{\gamma_c}\left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_P^2}\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)\right]\left[1 - \left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)^{1/2}\cos n\left(\sqrt{|k|}y\right)\right]}\right)^{1/2}. \tag{58}$$

Therefore the evolutionary form of the dilaton scalar field is obtained as

$$\phi(a) = \frac{2}{\lambda}\ln\left[e^{\frac{\lambda\phi(a_0)}{2}}\right.$$
$$\left.+\frac{\lambda}{\sqrt{2c'}}\int_{a_0}^{a}\frac{\mathrm{d}a}{Ha}\left(\frac{2 + b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right) + \frac{2}{3\gamma_c}\left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_P^2}\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)\right]\left[1 - \left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)^{1/2}\cos n\left(\sqrt{|k|}y\right)\right]}{4 + 3b^2\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right) + \frac{2}{\gamma_c}\left[1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_P^2}\left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)\right]\left[1 - \left(\frac{\Omega_\Lambda}{c^2\gamma_c}\right)^{1/2}\cos n\left(\sqrt{|k|}y\right)\right]}\right)^{1/2}\right]. \tag{59}$$

During the early inflation era, Eq. (59) yields

$$\phi(a) = \frac{2}{\lambda}\ln\left[e^{\frac{\lambda\phi(a_0)}{2}} + \frac{\lambda}{\sqrt{2c'}}\frac{1}{H}\left(\frac{2\Omega_\Lambda + b^2}{4\Omega_\Lambda + 3b^2}\right)^{1/2}\ln\left(\frac{a}{a_0}\right)\right], \tag{60}$$

where Ω_Λ is given by Eq. (43).

For the late-time universe, using Eq. (33) one can take the integral (59) as

$$\phi(a) = \frac{2}{\lambda}\ln\left\{e^{\frac{\lambda\phi(a_0)}{2}} + \frac{\lambda}{\sqrt{2c'}}\left(\frac{2+b^2-\frac{2}{3}\left(1-\frac{1}{c}\right)}{4+3b^2-2\left(1-\frac{1}{c}\right)}\right)^{1/2}\frac{\left[\left(\frac{a}{a_0}\right)^{1-\frac{1}{c}}-1\right]}{H_0(1-\frac{1}{c})}\right\}. \quad (61)$$

7 Conclusions

Here we considered the entropy-corrected version of the HDE model which is in interaction with DM in the non-flat FRW universe. The HDE model is an attempt for probing the nature of DE within the framework of quantum gravity [11]. We considered the logarithmic correction term to the energy density of HDE model. The addition of correction terms to the energy density of HDE is motivated from the LQG which is one of the promising theories of quantum gravity. Using this modified energy density, we obtained the EoS parameter for the interacting ECHDE. We established a correspondence between the interacting ECHDE model with the quintessence, tachyon, K-essence and dilaton scalar field models in the non-flat FRW universe. These correspondences are important to understand how various candidates of DE are mutually related to each other.

In the present case, the correspondence is established between ECHDE and various scalar field models of DE. We adopted the viewpoint that these scalar field models of DE are effective theories of an underlying theory of DE. Thus, we should be capable of using these scalar field models to mimic the evolving behavior of the interacting ECHDE and reconstructing the scalar field models according to the evolutionary behavior of the interacting ECHDE. We reconstructed the potentials and the dynamics of these scalar field models, which describe accelerated expansion of the universe, according to the evolutionary behavior of the interacting ECHDE model. We also obtained the explicit evolutionary forms of the corresponding scalar fields for the both of early inflation ($L = R_{\rm h} = H^{-1}=$ const.) and late-time acceleration ($L = R_{\rm h} \neq H^{-1}$ and $H \neq$ const.) phases. For late-time acceleration, L is dynamical and $\dot{L}$ will contribute in the above expressions and will yield the scalar potentials for the DE. This suggests that the vacuum energy that produced inflation at early cosmic epoch and the one driving late-time cosmic acceleration are fundamentally different. Hence the same scalar field moves in different potentials at different times.

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References

[1] A.G. Riess, et al., Astron. J. **116**, 1009 (1998);
S. Perlmutter, et al., Astrophys. J. **517**, 565 (1999);

P. de Bernardis, et al., Nature **404**, 955 (2000);
S. Perlmutter, et al., Astrophys. J. **598**, 102 (2003).

[2] T. Padmanabhan, Phys. Rep. **380**, 235 (2003);
P.J.E. Peebles, B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).

[3] E.J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006).

[4] P. Horava, D. Minic, Phys. Rev. Lett. **85**, 1610 (2000);
P. Horava, D. Minic, Phys. Rev. Lett. **509**, 138 (2001);
S. Thomas, Phys. Rev. Lett. **89**, 081301 (2002).

[5] G. 't Hooft, gr-qc/9310026;
L. Susskind, J. Math. Phys. **36**, 6377 (1995).

[6] A. Cohen, D. Kaplan, A. Nelson, Phys. Rev. Lett. **82**, 4971 (1999).

[7] W. Fischler, L. Susskind, hep-th/9806039.

[8] M. Li, Phys. Lett. B **603**, 1 (2004).

[9] K. Enqvist, M.S. Sloth, Phys. Rev. Lett. **93**, 221302 (2004);
Q.G. Huang, Y. Gong, J. Cosmol. Astropart. Phys. **08**, 006 (2004);
Q.G. Huang, M. Li, J. Cosmol. Astropart. Phys. **08**, 013 (2004);
Y. Gong, Phys. Rev. D **70**, 064029 (2004).

[10] E. Elizalde, S. Nojiri, S.D. Odintsov, P. Wang, Phys. Rev. D **71**, 103504 (2005);
X. Zhang, F.Q. Wu, Phys. Rev. D **72**, 043524 (2005);
B. Guberina, R. Horvat, H. Stefancic, J. Cosmol. Astropart. Phys. **05**, 001 (2005);
J.Y. Shen, B. Wang, E. Abdalla, R.K. Su, Phys. Lett. B **609**, 200 (2005);
B. Wang, E. Abdalla, R.K. Su, Phys. Lett. B **611**, 21 (2005).

[11] J.P.B. Almeida, J.G. Pereira, Phys. Lett. B **636**, 75 (2006);
B. Guberina, R. Horvat, H. Nikolic, Phys. Lett. B **636**, 80 (2006);
H. Li, Z.K. Guo, Y.Z. Zhang, Int. J. Mod. Phys. D **15**, 869 (2006);
X. Zhang, Phys. Rev. D **74**, 103505 (2006);
X. Zhang, F.Q. Wu, Phys. Rev. D **76**, 023502 (2007).

[12] L. Xu, J. Cosmol. Astropart. Phys. **09**, 016 (2009);
M. Jamil, M.U. Farooq, M.A. Rashid, Eur. Phys. J. C **61**, 471 (2009);
M. Jamil, M.U. Farooq, Int. J. Theor. Phys. **49**, 42 (2010);
A. Sheykhi, Class. Quant. Grav. **27**, 025007 (2010);
A. Sheykhi, Phys. Lett. B **682**, 329 (2010);
K. Karami, J. Fehri, Int. J. Theor. Phys. **49**, 1118 (2010);
K. Karami, J. Cosmol. Astropart. Phys. **01**, 015 (2010);
K. Karami, arXiv:1002.0431.

[13] R. Banerjee, B.R. Majhi, Phys. Lett. B **662**, 62 (2008);
R. Banerjee, B.R. Majhi, J. High Energy Phys. **06**, 095 (2008);
R. Banerjee, S.K. Modak, J. High Energy Phys. **05**, 063 (2009);
B.R. Majhi, Phys. Rev. D **79**, 044005 (2009);
S.K. Modak, Phys. Lett. B **671**, 167 (2009);
M. Jamil, M.U. Farooq, J. Cosmol. Astropart. Phys. **03**, 001 (2010);
M. Jamil, A. Sheykhi, M.U. Farooq, Int. J. Mod. Phys. D **19**, 1831 (2010);
H.M. Sadjadi, M. Jamil, arXiv:1002.3588;
S.W. Wei, Y.X. Liu, Y.Q. Wang, H. Guo, arXiv:1002.1550;
D.A. Easson, P.H. Frampton, G.F. Smoot, arXiv:1003.1528;
M. Jamil, A. Sheykhi, arXiv:1003.5043.

[14] H. Wei, Commun. Theor. Phys. **52**, 743 (2009).

[15] X. Zhang, Phys. Rev. D **74**, 103505 (2006);
X. Zhang, Phys. Lett. B **648**, 1 (2007);
J. Zhang, X. Zhang, H. Liu, Phys. Lett. B **651**, 84 (2007);
J. Zhang, X. Zhang, H. Liu, Eur. Phys. J. C **54**, 303 (2008);
X. Zhang, Phys. Rev. D **79**, 103509 (2009);
L.N. Granda, A. Oliveros, Phys. Lett. B **671**, 199 (2009).

[16] K. Karami, S. Ghaffari, J. Fehri, Eur. Phys. J. C **64**, 85 (2009);
K. Karami, J. Fehri, Phys. Lett. B **684**, 61 (2010);
K. Karami, M.S. Khaledian, F. Felegary, Z. Azarmi, Phys. Lett. B **686**, 216 (2010);
K. Karami, A. Abdolmaleki, Astrophys. Space Sci. **330**, 133 (2010);
K. Karami, A. Abdolmaleki, Phys. Scr. **81**, 055901 (2010);
K. Karami, A. Sorouri, Phys. Scr. **82**, 025901 (2010);
A. Sheykhi, M. Jamil, arXiv:1010.0385;
M. Jamil, E.N. Saridakis, J. Cosmol. Astropart. Phys. **07**, 028 (2010).

[17] J.P. Wu, D.Z. Ma, Y. Ling, Phys. Lett. B **663**, 152 (2008).

[18] A. Ali, M. Sami, A.A. Sen, Phys. Rev. D **79**, 123501, (2009).

[19] H.M. Sadjadi, M. Jamil, arXiv:1005.1483.

[20] C.L. Bennett, et al., Astrophys. J. Suppl. **148**, 1 (2003);
D.N. Spergel, Astrophys. J. Suppl. **148**, 175 (2003);
M. Tegmark, et al., Phys. Rev. D **69**, 103501 (2004);
U. Seljak, A. Slosar, P. McDonald, J. Cosmol. Astropart. Phys. **10**, 014 (2006);
D.N. Spergel, et al., Astrophys. J. Suppl. **170**, 377 (2007).

[21] Q.G. Huang, M. Li, J. Cosmol. Astropart. Phys. **08**, 013 (2004).

[22] T.P. Waterhouse, J.P. Zipin, arXiv:0804.1771.

[23] C.M. Paris, M. Visser, Phys. Lett. B **455**, 90 (1999).

[24] O. Bertolami, F. Gil Pedro, M. Le Delliou, Phys. Lett. B **654**, 165 (2007).

[25] H. Kim, H.W. Lee, Y.S. Myung, Phys. Lett. B **632**, 605 (2006);
M. Jamil, M.A. Rashid, Eur. Phys. J. C **56**, 429 (2008);
M. Jamil, M.A. Rashid, Eur. Phys. J. C **58**, 111 (2008);
M. Jamil, M.A. Rashid, Eur. Phys. J. C **60**, 141 (2009);
M. Jamil, F. Rahaman, Eur. Phys. J. C **64**, 97 (2009).

[26] S. Tsujikawa, M. Sami, Phys. Lett. B **603**, 113 (2004).

[27] S.D.H. Hsu, Phys. Lett. B **594**, 13 (2004).

[28] B. Wang, C.Y. Lin, E. Abdalla, Phys. Lett. B **637**, 357 (2006).

[29] A. Sen, Mod. Phys. Lett. A **17**, 1797 (2002);
N.D. Lambert, I. Sachs, Phys. Rev. D **67**, 026005 (2003);
M.U. Farooq, M.A. Rashid, M. Jamil, Int. J. Theor. Phys. **49**, 2278 (2010).

[30] M. Sami, et al., Phys. Rev. D **66**, 043530 (2002).

[31] G.W. Gibbons, Phys. Lett. B **537**, 1 (2002).

[32] A. Sen, J. High Energy Phys. **10**, 008 (1999);
E.A. Bergshoeff, M. de Roo, T.C. de Wit, E. Eyras, S. Panda, J. High Energy Phys. **05**, 009 (2000);
A. Sen, J. High Energy Phys. **04**, 048 (2002);
A. Sen, J. High Energy Phys. **07**, 065 (2002);
T. Padmanabhan, Phys. Rev. D **66**, 021301 (2002);
T. Padmanabhan, T.R. Choudhury, Phys. Rev. D **66**, 081301 (2002).

[33] R.J. Scherrer, Phys. Rev. Lett. **93**, 011301 (2004);
T. Chiba, S. Dutta, R.J. Scherrer, Phys. Rev. D **80**, 043517 (2009).

[34] R. Das, T.W. Kephart, R.J. Scherrer, Phys. Rev. D **74**, 103515 (2006).

[35] T. Chiba, T. Okabe, M. Yamaguchi, Phys. Rev. D **62**, 023511 (2000);
C. Armendáriz-Picón, V. Mukhanov, P.J. Steinhardt, Phys. Rev. Lett. **85**, 4438 (2000);
C. Armendáriz-Picón, V. Mukhanov, P.J. Steinhardt, Phys. Rev. D **63**, 103510 (2001).

[36] C. Armendáriz-Picón, T. Damour, V. Mukhanov, Phys. Lett. B **458**, 209 (1999);
J. Garriga, V. Mukhanov, Phys. Lett. B **458**, 219 (1999).

[37] M. Gasperini, F. Piazza, G. Veneziano, Phys. Rev. D **65**, 023508 (2002);
N. Arkani-Hamed, P. Creminelli, S. Mukohyama, M. Zaldarriaga, J. Cosmol. Astropart. Phys. **04**, 001 (2004);
E. Elizalde, S. Jhingan, S. Nojiri, S.D. Odintsov, M. Sami, I. Thongkool, Eur. Phys. J. C **53**, 447 (2008).